The previous chapter (Chapter 4) described the Generic Synthetic Aperture Ultrasound Imaging (GSAU), which is a direct copy of the Airborne Synthetic Aperture Radar. This type of synthetic aperture imaging is natural for the radar systems, since in most cases only a single antenna is available. In ultrasound scanners, on the other hand, all of the transducer elements are accessible at the same time, and there is no reason not to use them, unless some restrictions on complexity of the hardware are imposed. This is equivalent to having the aircraft at all imaging positions at the same time, giving much higher flexibility in the focusing algorithms as compared to the SAR systems.

Many researchers have tried to exploit this flexibility and a number of methods are described in the literature. Generally in ultrasound imaging two apertures can be defined: transmit and receive apertures. This chapter introduces a classification of the synthetic aperture algorithms depending on which of the two apertures is being synthesized. Three cases are possible: (1) synthetic receive aperture, (2) synthetic transmit aperture, and (3) synthetic transmit and receive apertures.

5.1 Synthetic receive aperture

5.1.1 Simple model

This type of synthetic aperture imaging was studied by the ultrasound group at Duke University and was presented in a series of papers by Nock and Trahey [69, 70, 71]. In 1994 Walker and Trahey presented some experimental results the IEEE Ultrasonics symposium [72]. One of the systems, the XTRA system, which is used for some of the measurements in the following, is also implemented as a synthetic receive aperture system (see Appendix J). In all of the listed cases the reason for using a synthetic receive aperture was to reduce complexity of the hardware. The complex part is usually assumed to be the receiver. If the system is digital and uses $N_{act}$ of the transducer elements at the same time, this means building $N_{act}$ receive channels. Each of the channels usually consists of analog amplifier, matched filter, analog to digital converter, and delay lines capable of dynamic focusing. The idea behind the receive synthetic aperture imaging is, that only one or some of the transducer elements can be used in receive. This results in a small active receive aperture. Changing (multiplexing) between different elements in receive, allows for a bigger receive aperture to be synthesized. The approach is illustrated in Figure 5.1. In transmit all of the elements focus along one line. In receive only one transducer element is active. Its data is memorized. Then the transmission is repeated in the same direction. Again only a single element is active in receive. The procedure is repeated...
Figure 5.1: Receive synthetic aperture imaging.

until all of the $N_{\text{adc}}$ transducer elements have been used in receive.

5.1.2 Performance

The performance is evaluated by the resolution, the achieved frame rate and the signal-to-noise ratio.

Resolution

The resolution of the system is determined by its $k$-space bandwidth. The spatial bandwidth is given by the effective aperture (see Section 3.7). Because of its nature, the $k$-space representation of the SRAU is equal to the $k$-space of the normal phased array imaging and so is the the resolution.

Frame rate

The frame rate is determined by the number of lines per image $N_l$, and the number of emissions $N_{\text{firings}}$ necessary to acquire a single line:

$$f_{fr \text{ SRAU}} = \frac{f_{pr f}}{N_l \cdot N_{\text{firings}}},$$  \hspace{1cm} (5.1)

where $f_{pr f}$ is the pulse repetition frequency.
The simple model assumed that the reception was performed only by a single element, i.e. \( N_{\text{firings}} = N_{\text{xdc}} \). It is possible that more than one element is active in receive, i.e. a small sub-aperture is active. In many applications only some of the transducer elements are used to beamform a line. Let the total number of elements used in the synthetic receive aperture be \( N_{\text{rcv}} \), and the number of active elements in receive be \( N_{\text{act}} \). If no element is used twice, then the total number of emissions per line is:

\[
N_{\text{firings}} = \left\lceil \frac{N_{\text{rcv}}}{N_{\text{act}}} \right\rceil
\]  

(5.2)

The frame rate becomes:

\[
f_{fr \text{ SRAU}} = f_{prf} \cdot \frac{N_{\text{act}}}{N_{\text{lines}} \cdot N_{\text{rcv}}}
\]

(5.3)

The synthetic receive aperture imaging method has a frame rate \( f_{\text{frame}} \) which is a \( N_{\text{firings}} \) times lower than the one of the phased array imaging:

\[
\frac{f_{fr \text{ SRAU}}}{f_{fr \text{ ph}}} = \frac{1}{N_{\text{firings}}}
\]

(5.4)

When \( N_{\text{act}} = N_{\text{rcv}} \) only one transmission is necessary per scan line. The method becomes equivalent to the normal phased or linear array imaging.

The frame rate for the generic synthetic aperture ultrasound, given in Chapter 4 is:

\[
f_{fr \text{ GSAU}} = \frac{f_{prf}}{N_{\text{rcv}}},
\]

(5.5)

The ratio between the frame rates between the generic synthetic aperture imaging and the synthetic receive aperture imaging is:

\[
\frac{f_{fr \text{ GSAU}}}{f_{fr \text{ SRAU}}} = \frac{N_{l} \cdot N_{\text{firings}}}{N_{\text{rcv}}}
\]

(5.6)

### Signal-to-noise ratio

The signal-to-noise ratio for the synthetic receive aperture imaging is as the signal-to-noise ratio for the phased array imaging. The gain in \( \text{SNR} \) compared to the \( \text{SNR} \) of of the signal of a single element is given by[65]:

\[
\frac{\text{SNR}}{\text{SNR}_0} = 10 \log N_{\text{xdc}}^3 \text{ dB},
\]

(5.7)

assuming that the number of transmitting elements is equal to the number of receiving elements, and is equal to the total number of transducer elements \( N_{\text{xdc}} \).

### 5.1.3 Implementation

The implementation of a synthetic receive aperture system is illustrated in Figure 5.2. In receive only a single element is multiplexed to the receive amplifier. The amplification can be changed as a function of time (TGC). The signal is digitized and stored in RAM. The implementation of
the RAM can be in two pages, each capable of storing one RF line of data. In order to maintain real-time processing, while the sampled data is fed into one of the pages, the data from the other is read and processed by the beamformer. The beamformer performs dynamic focusing.

The transmitter unit consists of a pulse generator, delay lines and amplifiers. The delay lines realize a fixed focus and their delays change only from line to line.

The role of the CONTROL is to synchronize the work of all the units, to notify the beamformer which receive channel is active and which line is being beamformed. It is also responsible for setting the delays in transmit.

5.2 Synthetic transmit aperture

The synthetic receive aperture imaging has the advantage over the generic synthetic aperture imaging of a higher signal-to-noise ratio and lower sidelobe level. It possesses one serious disadvantage, namely long acquisition time - from all of the methods considered so far, it is the slowest. A way to increase the frame rate is offered by the synthetic transmit aperture
5.2. Synthetic transmit aperture

ultrasound imaging. In the following we will consider a simple model and the performance of the method

5.2.1 Simple model

The simple model for the synthetic transmit aperture ultrasound imaging (STAU) is given in Figure 5.3.

In transmission only a single element is used. It creates a cylindrical wave (in the elevation plane the shape of the wavefront is determined by the height of the transducer and the presence/absence of an acoustic lens) which covers the whole region of interest. The received echo comes from all imaging directions, and the received signals can be used to create a whole image - in other words all of the scan lines can be beamformed in parallel. The created image has low-resolution because there is no focusing in transmit, and therefore in the rest of this report it is called a low-resolution image (LRI). After the first LRI, \( L_1(t) \), is acquired another element transmits and a second LRI, \( L_2(t) \), is created. After all of the transducer elements \( N_{dec} \) have transmitted, the low resolution images are summed and a high-resolution image (HRI) is created. In this case only \( N_{dec} \) emissions are necessary to create a high-resolution image. It is possible to use only some of the transducer elements to speed up the acquisition process \([1, 73, 74]\). The result is a sparse synthetic transmit aperture. The technique was thoroughly studied by Hazard and Lockwood \([75]\) who are currently developing a system for real-time synthetic transmit aperture imaging. The design of sparse arrays (synthetic and real) is considered
in greater detail in Chapter 8. The synthetic transmit aperture technique was used by Bae and colleagues [76] to increase the resolution of the acquired B-mode images using the standard focused transmissions. This approach will be more thoroughly considered in Chapter 7.

Let \( L_i(t) \) be a single low resolution line in the image, obtained after the emission with element \( i \in [1,N_{xdc}] \). The beamformation procedure for this line can be expressed as:

\[
L_i(t) = \sum_{j=1}^{N_{xdc}} a_{ij}(t) r_{ij}(t - \tau_{ij}(t)),
\]

where \( t \) is the time measured from the trigger of the emission, \( r_{ij}(t) \) is the signal received by the element \( j \). \( \tau \) and \( a \) are the applied delay and apodization coefficients, respectively. These two coefficients depend on which line in the image is being beamformed, and the time necessary for the wavefront to propagate from the transmitting element \( i \) to the focal point and back to the receiving element. Generally \( \tau \) and \( a \) are also functions of time in the case of dynamic focusing and apodization.

Figure 5.4 shows the geometry used to calculate the delays. The current focal point is \( \vec{x}_f \).

The time necessary for the emitted wave to propagate from the emitting element \( i \) to the focal point \( f \) and back to the receiving element \( j \) is given by:

\[
t_{ij} = \frac{1}{c} \left( |\vec{x}_f - \vec{x}_i| + |\vec{x}_f - \vec{x}_j| \right),
\]

where \( \vec{x}_i, \vec{x}_j \) and \( \vec{x}_f \) are the positions of the emitting, receiving and the focal point, respectively. The time that is necessary for a wave to propagate from the origin \( \vec{x}_c \) to the focal point \( \vec{x}_f \) and back is given by:

\[
t_{cc} = \frac{2}{c} |\vec{x}_f - \vec{x}_c|
\]

\[1\text{For notational completeness } L_i(t) \text{ must be } L_{il}(t), \text{ where } l \text{ is the line number. This, however, just makes the notations more complicated without having any influence on the results.}\]
The delay $\tau_{ij}$ necessary to apply on the signal received by the element $j$ after transmitting with element $i$ is:

$$\tau_{ij} = t_{ij} - t_{cc}$$ (5.11)

After the low-resolution scan lines have been all beamformed, the high-resolution ones, $H_i(t)$, can be beamformed by using a straight forward sum:

$$H_i(t) = \sum_{i=1}^{N_{xdc}} L_i(t)$$ (5.12)

Equation (5.12) shows the formation of a single scan line. One LRI $L_i(t)$ can be represented as a matrix, the columns of which are the low resolution scan lines $L_{il}(t)$, where $l$ ($1 \leq l \leq N_l$) is the index of the scan line. The same applies for the high resolution images $H(t)$. The operation (5.12) can then be expressed in a matrix form:

$$H(t) = \sum_{i=1}^{N_{xdc}} L_i(t).$$ (5.13)

The high resolution image is obtained by summing the low resolution images. This operation is illustrated in Figure 5.3.

### 5.2.2 Performance

The synthetic transmit aperture ultrasound imaging covers the whole $k$-space as the synthetic receive aperture imaging does. The resolution is therefore the same as for the case of synthetic receive aperture imaging. The number of emissions necessary to scan the image is:

$$N_{\text{firings}} = N_{\text{xdc}},$$ (5.14)

and the frame rate becomes:

$$f_{\text{frame}} = \frac{f_{\text{prf}}}{N_{\text{xdc}}}.\quad (5.15)$$

It gives the possibility for much faster imaging, especially if only some of the elements are used in transmit as discussed in Chapter 8. The signal-to-noise ratio and the penetration depths are, however, worser. The pressure amplitude is $N_{\text{xdc}}$ times smaller for the STA imaging, and the gain in signal-to-noise ratio is $[65]$:

$$\frac{\text{SNR}}{\text{SNR}_0} = 10\log\left(\frac{N_{\text{xdc}}^2}{2}\right).\quad (5.16)$$

This signal-to-noise ratio is mostly “academic”. Due to scattering and attenuation, after some depth there is not enough signal left in order to cause an echo detectable by the receiver. Means to increase the transmitted power are needed, and these include the use of virtual ultrasound sources (see Chapter 7) and spatial and temporal encoding (see Chapter 9).

### 5.3 Synthetic transmit and receive apertures

Figure 5.5 (taken from [65]) shows different combinations of active transmit and receive elements. The use of a single element in both, transmit and receive is what in this thesis is called
Chapter 5. Variations of SAU

Figure 5.5: Active transmit/receive element combinations for different types of synthetic aperture ultrasound imaging. Each row corresponds to one transmit event. The transmission is done with the element whose index is equal to the row number. The filled squares symbolize an active element in receive.

generic synthetic aperture imaging (top left plot). Using all of the elements in receive is called synthetic transmit aperture imaging (top right plot). The cases in which more than one elements (but not all) are used in receive are labeled as synthetic transmit and receive imaging (the bottom row). These types of imaging have been the topic of several papers \[67, 77, 78, 79]\) because of supposedly simple receiver design.

The bottom row of Figure 5.5 shows two different cases for transmit and receive synthetic aperture imaging. In the first case the elements that are neighboring to the transmit element are used in receive. Using neighboring elements fills in the gaps in the effective aperture (see Figure 4.5 on page 48) thus reducing the grating lobes level. The second case (see the bottom right plot in Figure 5.5) is supposed to generate all combinations of spatial frequencies (determined by the transmit receive pairs) without creating the same spatial frequency twice. For example transmitting with element \(i\) and receiving with element \(j\) is the same as transmitting with element \(j\) and receiving with element \(i\). The effective apertures of the four methods and their two-way radiation patterns are illustrated in Figure 5.6. The transducer pitch is assumed to be half a wavelength \((\lambda/2)\), and the elements have omnidirectional radiation pattern. The radiation patterns were obtained using the Fourier relation between them and point-spread function at the focus. The redundancy in the effective aperture caused by the multiple receive elements results in decreased side and grating lobe levels, and in an increase in signal-to-noise ratio.
5.3. Synthetic transmit and receive apertures

Figure 5.6: The effective apertures and the two-way radiation patterns of four variations of synthetic aperture imaging. The distance between the elements is $d_x = \lambda/2$ and the size of the elements is $w \to 0$.

### 5.3.1 Performance

The frame rate is determined by the number of emissions and the number of parallel receive beamformers. Assuming that all of the lines in the image can be beamformed in parallel, then the frame rate is:

$$f_{fr \, STRAU} = \frac{f_{prf}}{N_{xdc}}. \quad (5.17)$$

The signal-to-noise ratio and the penetration depth are worse than those of the SRA and STA imaging modalities. The gain in signal-to-noise ratio compared to the signal-to-noise ratio of the signal received by a single element is:

$$\frac{SNR}{SNR_0} = 10\log(N_{xdc}N_{rcv}), \quad (5.18)$$

where $N_{rcv}$ is the number of receive elements used at every reception and $N_{xdc}$ is the total number of elements.